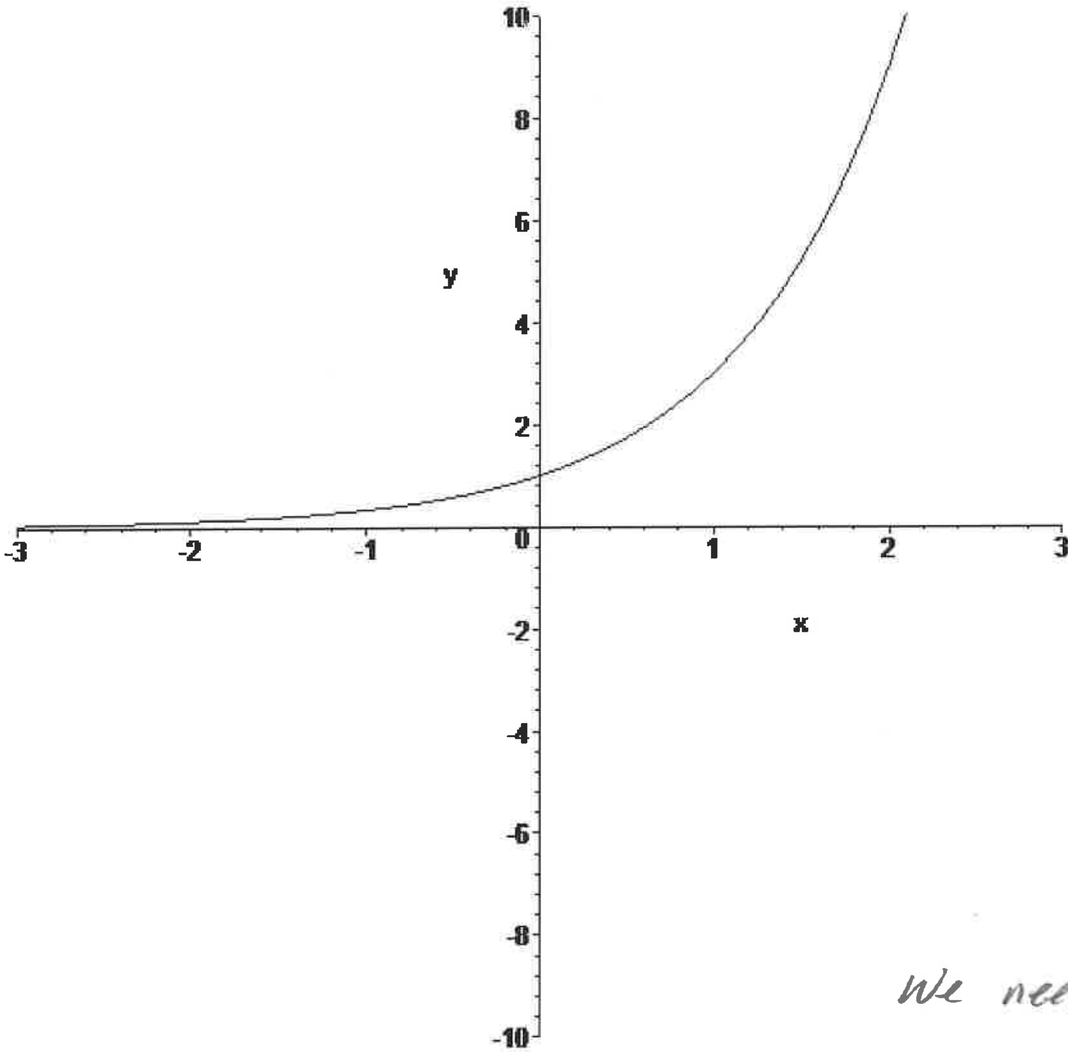
Logarithmic Functions

The graph of the function $f(x) = 3^x$ is given below:



We need an inverse of the function f

Solve the equation $3^x = 9$ $\quad x = 2$

Solve the equation $3^x = \frac{1}{3}$ $\quad x = -1$

Solve the equation $3^x = \sqrt{3}$ $\quad x = \frac{1}{2}$

Solve the equation $3^x = 7$ $\quad x \approx 1.771$

Solve the equation $3^x = 26$ $\quad x \approx 2.966$

Definition:

If b and a are positive real numbers then $log_b a$ is the number you would raise b by to get a. In other words, $log_b a$ is the unique solution to the equation $b^x = a$.

How do you interpret the following expressions:

$log_2 8$ $= x$, $2^x = 8$, $x = 3$

$log_2 \sqrt{2}$ $= x$, $2^x = \sqrt{2}$, $x = \frac{1}{2}$

$log_2 10$ $= x$, $2^x = 10$, $x \approx 3.322$

$log_5 71$ $= x$, $5^x = 71$, $x \approx 2.649$

$\left.\begin{array}{l} \\ \end{array}\right\}$ graph + trace

$log_2 0$ $= x$, $2^x = 0$, no solution

$log_3(-2)$ $= x$, $3^x = -2$, no solution

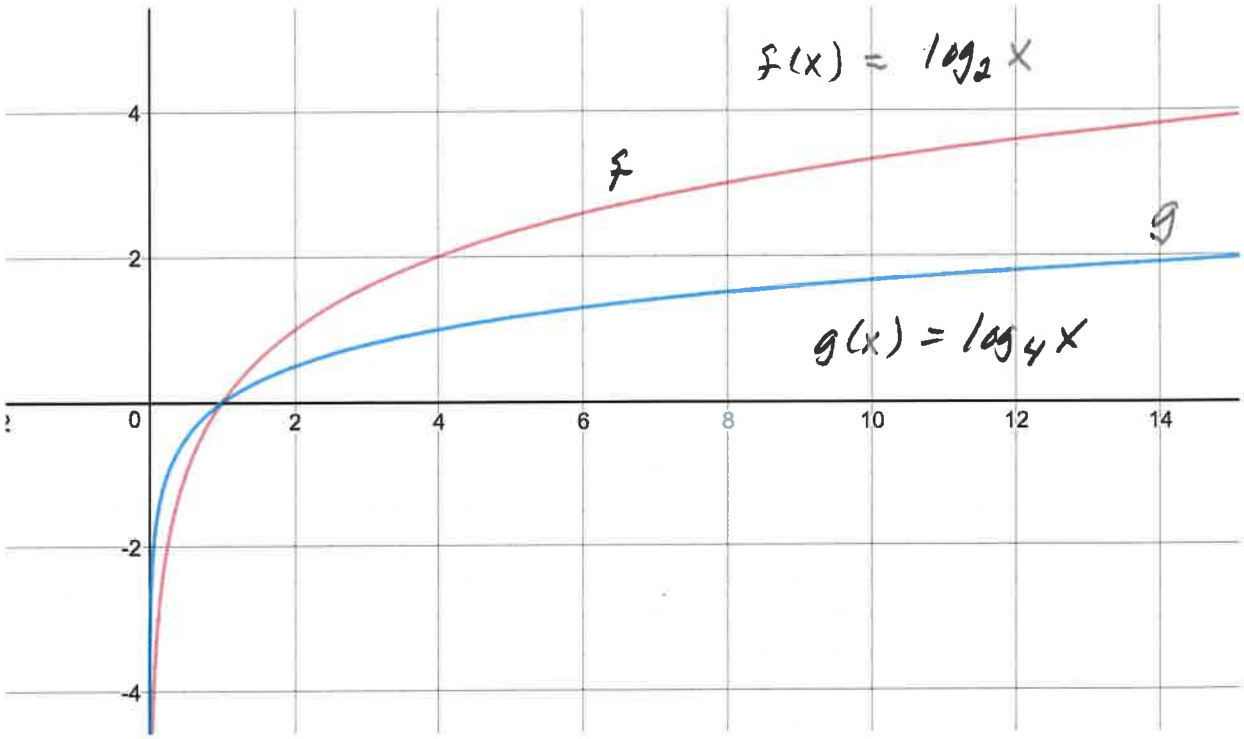
The exponential equation $a^x = y$ is equivalent to the logarithmic equation $log_a y = x$

$$a^x = y \quad \Leftrightarrow \quad log_a y = x$$

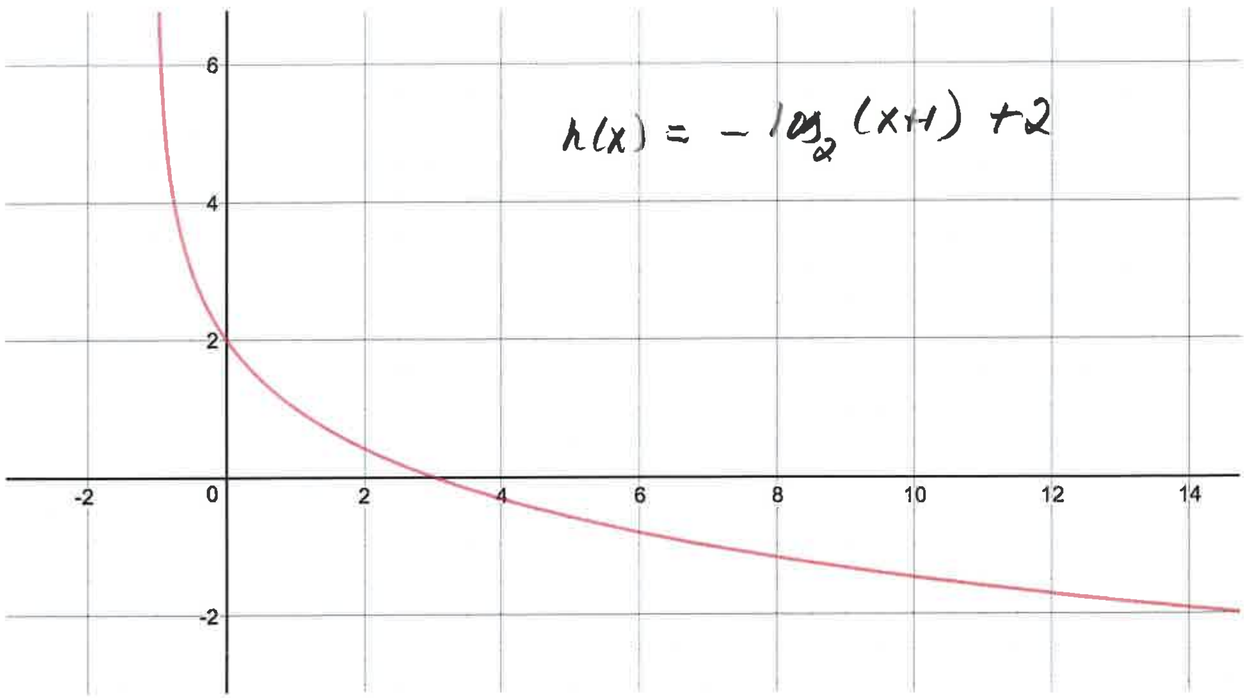
$$2^3 = 8 \quad \Leftrightarrow \quad log_2 8 = 3$$

$$4^x = 64 \quad \Leftrightarrow \quad log_4 64 = x$$

$$7^x = 29 \quad \Leftrightarrow \quad log_7 29 = x$$



$f(x) = \log_2 x$
f
g
$g(x) = \log_4 x$
4
2
0
-2
-4
2
4
6
8
10
12
14



$h(x) = -\log_2(x+1) + 2$
6
4
2
0
-2
-2
2
4
6
8
10
12
14

Basic Logarithm Properties :

$log_b 1 = 0$ (because $b^0 = 1$)

$lob_b b = 1$ (because $b^1 = b$)

$log_b(b^x) = x$

$b^{log_b x} = x$

Simplify the following expressions:

$log_2 \frac{1}{16} = log_2 2^{-4} = -4$

$log_5(5\sqrt{5}) = log_5 5^{(1+\frac{1}{2})} = 1.5$

$log_{16} 2 = log_{16} (16)^{\frac{1}{4}} = \frac{1}{4}$

The common logarithm and the natural logarithm:

$log_e x$ is called the natural log and is always written $ln\,x$.

$log_{10} x$ is called the common logarithm and is usually written as $log\,x$

Simplify the following if possible or give a 2 decimal approximation using a calculator if necessary:

$ln\,1$ $= 0$

$ln\,(e^3)$ $= 3$

$ln\,\frac{1}{e^2}$ $= ln(e^{-2}) = -2$

$ln\,40$ ≈ 3.6889

$ln\,(-3)$ Not defined

$log\,100$ $= log_{10} 100 = log_{10} 10^2 = 2$

$log\,0.0001$ $= log_{10} 10^{-4} = -4$

$log\,80$ ≈ 1.9031

$log\,0$ Not defined